Erratum: Optimal stopping time problem in a general framework

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The proof of the second point of Proposition B11 given in the Appendix of Kobylanski and Quenez (2012) ([1]) is only valid in the case where the reward process (ϕ_t) is right-continuous. We thus give below the proof in the case where (ϕ_t) is only right-upper-semicontinuous.

Proof of the second point of Proposition B11 in the general case:
The proof of the second point is based on Proposition B7 in [1] and on some analytic arguments similar to those used in the proof of Theorem D13 in Karatzas and Shreve (1998) ([2]). Without loss of generality, we can suppose that for each ω, the map $t \mapsto v_t^+(\omega)$ is right-continuous, the map $t \mapsto \phi_t(\omega)$ is right-upper-semicontinuous, and $t \mapsto A_t^c(\omega)$ is continuous.
Let us denote by $\mathcal{J}(\omega)$ the set on which the nondecreasing function $t \mapsto A_t^c(\omega)$ is "flat":

$$\mathcal{J}(\omega) := \{t \in]0, T[\,,\ \exists \varepsilon > 0 \text{ with } A^c_{t-\varepsilon}(\omega) = A^c_{t+\varepsilon}(\omega)\}$$

The set $\mathcal{J}(\omega)$ is clearly open and hence can be written as a countable union of disjoint intervals: $\mathcal{J}(\omega) = \cup_i]\alpha_i(\omega), \beta_i(\omega)[$. We consider

$$\hat{\mathcal{J}}(\omega) = \cup_i [\alpha_i(\omega), \beta_i(\omega)[= \{t \in [0, T[\,,\ \exists \varepsilon > 0 \text{ with } A^c_t(\omega) = A^c_{t+\varepsilon}(\omega)\}.$$

The nondecreasing function $t \mapsto A_t^c(\omega)$ is "flat" on $\hat{\mathcal{J}}(\omega)$, hence $\int_0^T \mathbb{I}_{\hat{\mathcal{J}}(\omega)} dA_t^c(\omega) = \sum_i (A^c_{\beta_i(\omega)}(\omega) - A^c_{\alpha_i(\omega)}(\omega)) = 0$.
We next show that for almost every ω, $\mathcal{H}^c(\omega) \subset \hat{\mathcal{J}}(\omega)$, which clearly provides the desired result.
Let us denote by $\mathbb{Q}$ the set of rationals. By Proposition B7 in [1] applied to constant stopping times $\theta := t$, where $t \in \mathbb{Q} \cap [0, T[$, it follows that for a.e. ω,

$$\{t \in \mathbb{Q} \cap [0, T[\text{ s.t. } v_t(\omega) > \phi_t(\omega)\} \subset \hat{\mathcal{J}}(\omega). \tag{1}$$

Let us now show that the desired inclusion

$$\mathcal{H}^c(\omega) = \{t \in [0, T[\text{ s.t. } v_t(\omega) > \phi_t(\omega)\} \subset \hat{\mathcal{J}}(\omega)$$

holds for a.e. ω.

To this purpose, we note that for each ω, $\phi_t(\omega) \geqslant \limsup_{s \to t^+} \phi_s(\omega)$ for each t, and $v_t(\omega) = v_t^+(\omega)$ for each $t \in \mathcal{H}^c(\omega)$, since $v_t = \phi_t \vee v_t^+$ (cf. equation (B2) in [1]). Hence, for each ω, $\mathcal{H}^c(\omega) \subset \mathcal{K}(\omega)$, where $\mathcal{K}(\omega) := \{t \in [0, T[\text{ s.t. } v_t^+(\omega) > \limsup_{s \to t^+} \phi_s(\omega)\}$. It is thus sufficient to show that for a.e. ω,

$$\mathcal{K}(\omega) = \{t \in [0, T[\text{ s.t. } v_t^+(\omega) > \limsup_{s \to t^+} \phi_s(\omega)\} \subset \hat{\mathcal{J}}(\omega).$$

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Fix ω such that (1) holds and fix $t \in \mathcal{K}(\omega)$. Since $v_t^+(\omega) > \limsup_{s \to t^+} \phi_s(\omega)$ and since the map $t \mapsto v_t^+(\omega)$ is right continuous, there exists a non increasing sequence of rationals $t_n(\omega) \in \mathbb{Q} \cap [0, T[$ such that $t = \lim_{n \to \infty} \downarrow t_n(\omega)$ with $v_{t_n(\omega)}^+(\omega) > \phi_{t_n(\omega)}(\omega)$ for each n. Since $v_.(\omega) \geqslant v_.^+(\omega)$, it follows that for each n,

$$t_n(\omega) \in \{t \in \mathbb{Q} \cap [0, T[\text{ s.t. } v_t(\omega) > \phi_t(\omega)\} \subset \hat{\mathcal{J}}(\omega),$$

where the last inclusion corresponds to (1). Using the equality $\hat{\mathcal{J}}(\omega) = \cup_i [\alpha_i(\omega), \beta_i(\omega)[$ and the fact that $t = \lim_{n \to \infty} \downarrow t_n(\omega)$, we derive that there exist i and n_0 (which both depend on ω) such that for each $n \geqslant n_0$, $t_n(\omega) \in [\alpha_i(\omega), \beta_i(\omega)[$. It follows that the limit $t \in [\alpha_i(\omega), \beta_i(\omega)[$, which gives that $t \in \hat{\mathcal{J}}(\omega)$. Hence, the inclusion $\mathcal{H}^c(\omega) \subset \hat{\mathcal{J}}(\omega)$ is proven, which ends the proof of the second point. $\square$

References

[1] Kobylanski, M. and Quenez, M.-C. (2012). Optimal stopping time problem in a general framework, *Electron.J.Probab.* **17**, No.72, 1-28.

[2] Karatzas I. and S. E. Shreve (1998), *Methods of mathematical finance*, Applications of Mathematics (New York), 39, Springer, New York.

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